Teaching physics with Hubble's law and dark matter

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(Received 14 September 2011; accepted 24 January 2012)

Physics instructors can enrich, enliven, and enhance their courses with conceptually rich cosmology content. In this paper, we specifically discuss how instructors can integrate lessons on Hubble's law (as it relates to the expansion of the universe and dark energy) and spiral galaxies' rotation curves (as they relate to the presence of dark matter) into an introductory, college-level course on mechanics. These cosmology topics intersect with the content of introductory physics in a number of areas, such as students' abilities to read and interpret graphs and their conceptual understandings of both kinematics and dynamics. Throughout this paper, we draw upon the results from, and research-validated curricula informed by, physics and astronomy education research. In particular, we feature the results from a national study we recently completed with introductory college-level general education astronomy students on the teaching and learning of cosmology.
[DOI: 10.1119/1.3684469]

I. INTRODUCTION

How can we use the results of modern astrophysics to enrich, enliven, and enhance undergraduate physics instruction? Answering this question is the goal of multiple instructors, astrophysics researchers, developers of curricula, and members of the astronomy education research (AER) and physics education research (PER) communities. Their efforts are being organized with topical workshops, special sessions at national meetings, and this theme issue of the *American Journal of Physics*, all leading up to the June 2012 Gordon Research Conference on "Astronomy's Discoveries and Physics Education."[1] By moving beyond traditional physics content, such as inclined planes and Atwood machines, and by infusing the physics curriculum with content based on real problems scientists are trying to solve, we hope to motivate and inspire physics students and give them a better appreciation for the discovery nature of science. Research suggests that by enlivening instruction with interesting and useful applications, we can improve students' interest in science and increase the number of students who complete a degree in science.[2]

Cosmology is one area of modern astrophysics that is full of interesting and useful applications of physics. Theoretical and technological advances have allowed cosmology researchers to address topics such as the beginning, age, composition, evolution, and fate of the universe. We now have multiple lines of evidence that point to a concordant cosmological model.[3–5] This model permits cosmologists to make detailed descriptions of and predictions about our universe—leading some to declare that we are in an era of "precision cosmology."[6]

In this paper, we discuss how instructors can use content and contexts drawn from modern cosmology to enhance their introductory physics courses. We specifically focus on content that is appropriate for an introductory, college-level, calculus-based physics course on mechanics taken by STEM (science, technology, engineering, and mathematics) majors (hereafter "Phys 101"). As such, we will avoid presentations and topics that require physical knowledge and mathematical formalism beyond what the average freshman physics student possesses. Since many introductory physics students also take introductory calculus simultaneously, we cannot assume their knowledge of calculus extends much beyond ordinary derivatives.

Our purpose in this paper is not to present a detailed lesson plan for a section on cosmology at the end of the semester, nor is it to provide a comprehensive, mathematical treatment of cosmology topics (which may be found elsewhere; see, for example, Refs. 4, 7–11). Instead, we look at how a Phys 101 course that incorporates content drawn from cosmology might be informed by findings from AER on which concepts are particularly challenging for students and how research-supported activities can improve students' understandings of these conceptually challenging topics. We draw heavily upon prior work in AER and PER. Prior research shows that students are often able to solve traditional end-of-chapter problems while maintaining naïve ideas about the topics covered by those problems.[12,13] Additionally, students in courses that devote more time to building conceptual understanding typically perform at least as well on traditional problem-solving exercises as their peers in traditionally taught courses.[13,14] We, therefore, focus on interactive, collaborative activities that research shows improve students' conceptual understandings. These activities include Think-Pair-Share (similar to Mazur's "Peer Instruction"), Ranking Tasks, and tutorials.[13,15–19] Many of the activities we discuss were developed as a result of our recent national study on the conceptual and reasoning difficulties students enrolled in a college-level, general education, introductory astronomy course (hereafter Astro 101) experience with cosmology.[20] These activities can strengthen students' conceptual understandings and prepare them for more advanced cosmology problems (such as those contained in Refs. 8–11).

We first discuss Hubble's law as it relates to the expansion of the universe and the evidence for dark energy. We then discuss the rotation curves of spiral galaxies and how they provide evidence for dark matter. Throughout our discussion, we describe how and where Phys 101 instructors can integrate these topics into their courses.

II. HUBBLE'S LAW

Velocity, acceleration, and other kinematic concepts are among the first topics taught in many Phys 101 courses. Multiple PER studies show that these topics present numerous conceptual and reasoning difficulties for students. For example, many students have trouble interpreting velocities and accelerations as ratios, fail to distinguish between velocity and acceleration, conceive of acceleration as a change in velocity per unit distance, think two objects have the same velocity and/or acceleration when they are at the same position, and struggle to interpret graphs of kinematic quantities.[21–23] While numerous research-validated activities have been developed to help students overcome these naïve ideas (e.g., Refs. 16 and 18), instructors teaching kinematics may search for other ways to counter these ideas and enhance and extend their students' understandings of kinematics. The cosmology topics of Hubble's law and Hubble plots offer a fascinating context for teaching kinematics.

Hubble's law encapsulates the observation that distant galaxies have recessional velocities that are proportional to their distances from us. Hubble's law may be written as

$$v = H(t) \cdot d, \tag{1}$$

where v represents the recessional velocity of a galaxy, d represents the galaxy's distance from an observer, and $H(t)$ is the Hubble parameter, a proportionality constant that is the same for all galaxies but that varies with time. The current value of $H(t)$, known as Hubble's constant H_0, is established by recent observations to be $H_0 = 74.2 \pm 3.6$ km/s/Mpc.[24]

Many instructors begin an introduction to Hubble's law by discussing how we infer recessional velocities v from galaxies' observed redshifts, and how we infer distances d from observations of the relative brightnesses of objects within galaxies. Since astronomers use many different techniques to determine distances, and because recessional velocities are related to galaxies' redshifts through a non-trivial general relativistic relationship (Ref. 25, p. 99), many physics instructors may find these details too far afield for their Phys 101 course. Our instructional approach to Hubble's law, therefore, does not focus on how one obtains v and d from observations. Instead, we focus on how one can use Hubble's law to improve and extend students' understandings of relevant kinematics concepts.

Graphs of Eq. (1), known as Hubble plots, offer a useful way to use Hubble's law to teach kinematics. The slope of the curve of a Hubble plot (i.e., the value of $H(t)$ for a particular time t) is a measure of the expansion rate of the universe at time t. Figure 1 shows an example of a Hubble plot with a constant Hubble parameter and, consequently, a constant expansion rate. Note that Fig. 1, along with many subsequent graphs in this paper, is a sketch of the relationship we are talking about. Though such schematic representations lose

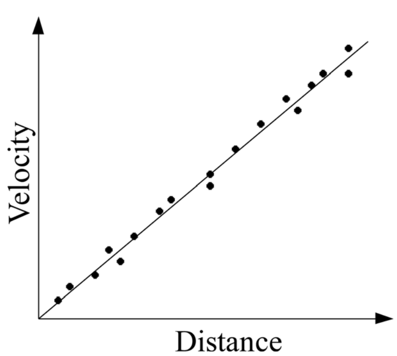


Fig. 1. A schematic Hubble plot for a universe with a constant Hubble parameter.

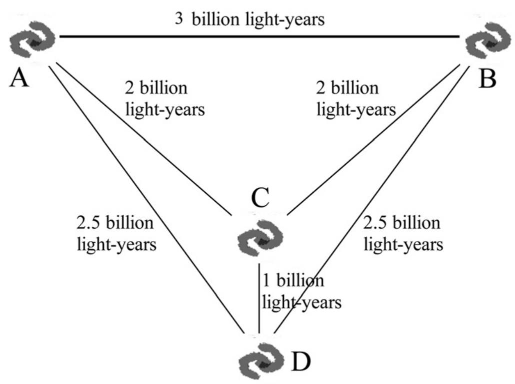
some of the richness of graphs of actual data (with their attendant uncertainties), they enable students to focus their attentions on the critical conceptual ideas we are trying to teach. By using such heuristic representations, we are following the well-blazed path laid out by the previous education researchers, such as the Physics Education Group at the University of Washington, whose tutorials and homework exercises on kinematics also make use of sketches of graphs of kinematic quantities.[18,26] Throughout this paper, the reader should assume that all axes on these schematic plots are on a linear, rather than a logarithmic, scale.

Students should investigate Hubble plots, such as Fig. 1, early on in order to force them to reason about how the information presented in a Hubble plot relates to aspects of the real physical universe. Hubble plots are particularly challenging for students to interpret since most of the other kinematics graphs they encounter use time, not distance, as the x-axis variable. Results from our national study revealed many difficulties students experience with regards to Hubble plots. For example, many students think that a universe with a constant expansion rate should have a Hubble plot with a horizontal line, indicating a universe in which the recessional velocities of all galaxies are the same for all distances. Students also have trouble correctly interpreting a Hubble plot with a constant but nonzero slope (typically incorrectly thinking that this indicates a universe with an increasing or decreasing expansion rate). By far the greatest conceptual and reasoning difficulties are observed when students are asked to interpret Hubble plots that are nonlinear (on which we say more below). Students also have a hard time understanding that the origin on a Hubble plot represents our location in the universe and that a similar Hubble plot could be generated from any location within the universe.[20]

Students should also perform a dimensional analysis of the Hubble parameter $H(t)$. This dimensional analysis reveals that $H(t)$ has dimensions of 1/time. This yields an important insight into our understanding of the universe since it shows that the slope of a line on a Hubble plot is inversely related to the age of the universe. During our national study, we found that many students struggle to understand how changing the slope of a line on a Hubble plot changes our estimation of the age of the universe.[20]

With these difficulties in mind, what might a Phys 101 instructor do to incorporate Hubble's law and Hubble plots into his or her course? Because velocity versus distance graphs are not as common in Phys 101 as velocity versus time graphs, instructors may need to begin by providing their students with activities that illustrate the physical situation depicted by Hubble plots. One such activity that instructors can use is the "Hubble's Law" *Lecture-Tutorial*, which we developed and validated as part of our national study of Astro 101 students' learning difficulties with cosmology.[20] Figure 2 shows the first question of this *Lecture-Tutorial*. Students must redraw a picture of four galaxies after the universe doubles in size. Most students intuitively and correctly double all the distances between the galaxies. This question forms the basis for subsequent questions that guide students to the realization that, in a universe with a constant rate of expansion, the recessional velocities of distant galaxies are linearly related to their distances from us: Farther galaxies appear to move away at faster velocities than closer galaxies (see Fig. 1). This can be used as part of a lesson that directly addresses the common pre-instruction belief of many students that the Hubble plot of a universe with a constant

Consider the small section of the universe containing four galaxies (A-D), shown in the figure below. The distances between each galaxy are also shown.



1) Imagine that this section of the universe doubles in size over time due to the expansion of the universe. Draw what the above section of the universe would look like after it doubles in size. Be sure to identify the new distances between the galaxies.

Fig. 2. The first question of the "Hubble's Law" *Lecture-Tutorial*.

expansion rate should show a line that represents a constant velocity with increasing distance.

Our analysis of student responses from our national study suggests that many students also struggle to understand the significance of the origin on a Hubble plot. We found many cases in which students chose and/or drew, for a given physical situation, a Hubble plot with a line that did not intersect the origin.[20] Since the origin represents our location in the universe, this suggests that many students do not understand that the velocities and distances on Hubble plots are measured relative to our location. Since relative motion is an important topic covered by many units on kinematics, Hubble plots thus provide another opportunity to reiterate this important idea. Figure 3 shows one activity instructors can use to probe the idea of one's "origin" for observations and the subsequent relative motions one will measure. Students must use the relative velocities and distances shown in a Hubble plot to construct new Hubble plots for the observers in Galaxies 3 and 5.

Once students have developed the conceptual foundations necessary to interpret the Hubble plot for a universe undergoing constant expansion, they should be asked to reason about the role time plays in interpreting the age and expansion rate of a constantly expanding universe. This can be tricky for students, since, as mentioned earlier, time is not one of the variables shown on either the y- or x-axes. This is why, for example, many students struggle to connect the idea of the expansion rate to the age of the universe. Figure 4 shows a series of Ranking Tasks that help students to connect the slope of a Hubble plot to the universe's expansion rate and age. In the Ranking Tasks in Fig. 4, students must first rank the expansion rates of three hypothetical universes based on their Hubble plots. They are then asked to rank the average distances between galaxies in these universes from smallest to largest. These two Ranking Tasks provide students with the scaffolding they need to rank the three universes based on their ages. Having students explicitly reason about these quantities together is a critical step before

Figure A shows a Hubble plot constructed by observers on Earth. The velocities and distances of five galaxies (1-5) are shown. Use the blank graph provided in Figure B to sketch the Hubble plot that observers in Galaxy 5 would construct. Label the locations of Galaxies 1-4 and the Milky Way Galaxy on Figure B. Use the blank graph provided in Figure C to sketch the Hubble plot that observers in Galaxy 3 would construct. Label the locations of Galaxies 1, 2, 4, 5 and the Milky Way Galaxy on Figure C.

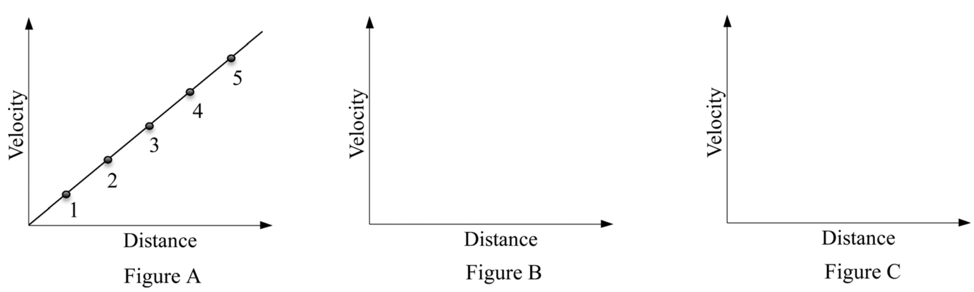
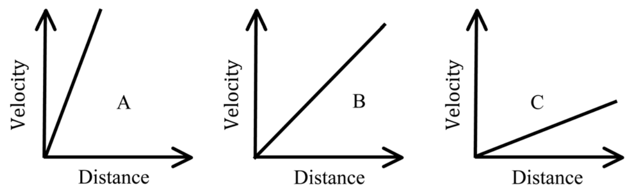


Fig. 3. A graphing activity that requires students to reason about how to construct Hubble plots while taking into account the observers location and how the recessional velocities of galaxies are relative to their distance from the observer.

Description: The following ranking tasks refer to the series of Hubble plots (A-C) shown below. Each Hubble plot depicts a different possible universe. Use the information provided in these plots to reason through each ranking task provided below.



A. Ranking Instructions: Rank the expansion rate (from fastest to slowest) that each universe would be experiencing.

Ranking Order:

Fastest ______ , ______ , ______ Slowest
Or, the expansion rate is the same for each universe ______ (indicate with a check mark).

Carefully explain your reasoning for ranking this way:

____

____

B. Ranking Instructions: Rank the average distances (from smallest to largest) between the galaxies in each universe for the particular case in which they all started at the same time and have been in existence for the same amount of time.

Ranking Order:

Smallest ______ , ______ , ______ Largest
Or, the average distances are the same for each universe ______ (indicate with a check mark).

Carefully explain your reasoning for ranking this way:

____

____

C. Ranking Instructions: Now rank the age (from oldest to youngest) of each universe based on the amount of time it would have taken each universe to reach a particular size.

Ranking Order:

Oldest ______ , ______ , ______ Youngest
Or, the age is the same for each universe ______ (indicate with a check mark).

Carefully explain your reasoning for ranking this way:

____

____

Fig. 4. A sample Ranking Task.

students move to more advanced problems (such as those in Ref. 9) dealing with non-constant expansion (see Ref. 27 for more information on pedagogical approaches to this topic).

At some point, instructors should present students with more complicated cases than constant expansion. For example, what if the universe was constantly contracting? What would the Hubble plot for the universe look like then? Figure 5 shows a version of this question that is appropriate for a Think-Pair-Share activity. This activity forces students to reason about the meaning of negative quantities on graphs, which research shows students struggle with, both in terms of traditional kinematics content and in terms of Hubble plots.[20,21] To choose the correct answer, students must reason that in a constantly contracting universe, farther galaxies are approaching us at faster velocities than closer galaxies. Since motion toward us is represented by negative values, the correct answer must be B. The other options represent popular incorrect choices frequently selected by students.[20] Options A, C, and D all show some aspect of the graph to be negative, be it the slope and/or the velocity. Students who do not understand that the slope must be constant, negative, and

Which of the following Hubble plots depicts a universe that is contracting at a constant rate?

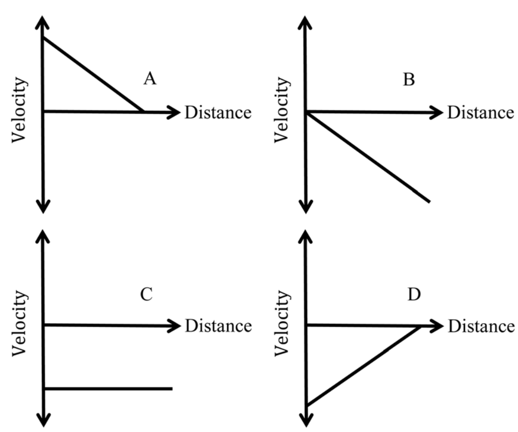


Fig. 5. A question, which may be used as a Think-Pair-Share activity, that makes students reason about the meaning of negative quantities on Hubble plots.

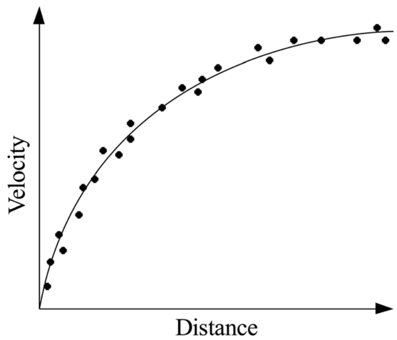


Fig. 6. A schematic Hubble plot for a universe with a Hubble parameter that changes with time.

non-zero and/or who do not understand that the line must pass through the origin will be drawn to one or more of these incorrect options.

Up to this point, we have only dealt with cases in which the Hubble parameter is constant. But what if the expansion rate is not constant, as recent observational evidence suggests?[28,29] Many students (and instructors) incorrectly reason that the Hubble plot in Fig. 6 depicts a universe whose expansion rate decreases with time. The reasoning error here is understandable as we are accustomed to reading a graph from left to right. Doing so, we notice a decreasing slope. The problem is that farther distances actually correspond to times that are further in the past (i.e., earlier in the history of the universe). So distances represented farther to the right on the horizontal axis actually represent events further back in time. As one moves from right to left in Fig. 6 (i.e., from the past toward the present), the slope, and hence the expansion rate, increases. Questions that probe this line of reasoning were among the most difficult for students to successfully defend with their written responses in our study.[20] Figure 7 shows a series of questions from the "Hubble's Law" *Lecture-Tutorial* that were designed to provide students with the necessary scaffolding to correctly interpret this graph depicting a universe with a changing expansion rate.

Questions on the changing expansion rate may make appropriate "end-of-chapter" estimation tasks for students. For example, instructors can provide their students with a Hubble plot for a universe with a non-constant expansion rate and ask their students to estimate the expansion rate (via the slope of the graph) at various locations (times).

With all this discussion of understanding and estimating changing expansion rates, let us not lose sight of why this information is important: The fact that our universe's expansion rate is accelerating provides important scientific evidence for the existence of dark energy.[27] Instructors and curricula should emphasize this idea since it yields a profound insight into our ignorance regarding the composition of the real universe in which we live. Though Phys 101 students are not yet

15) Parts a-h all refer to Figure 6. Draw or write the additional information on Figure 6 as instructed:

   a) Draw a circle around the galaxies from which we receive light that was emitted closest to our present time.

   b) Draw a square around the galaxies from which we receive light that was emitted furthest from our present time.

   c) Write the letter C, and draw an arrow to, the galaxies that are moving away from us with the fastest velocities.

   d) Write the letter D, and draw an arrow to, the galaxies that are moving away from us with the slowest velocities.

   e) Write the letter E, and draw an arrow to the graph, where it has the steepest slope.

   f) Write the letter F, and draw an arrow to the graph, where it has the flattest slope.

   g) Write the letter G, and draw an arrow to, the portion of the graph that corresponds with the fastest expansion rate.

   h) Write the letter H, and draw an arrow to, the portion of the graph that corresponds with the slowest expansion rate.

16) Based on the Hubble plot shown in Figure 6, would you say that the expansion rate of the universe is constant or changing with time? Explain your reasoning.

17) Based on the Hubble plot in Figure 6, is the expansion rate represented by the motion of galaxies far away from us faster than, slower than, or the same as the expansion rate represented by the motions of nearby galaxies? Explain your reasoning.

18) Based on the Hubble plot in Figure 6, is the expansion rate of the universe increasing or decreasing as time goes on? Explain your reasoning.

Fig. 7. A series of questions from the "Hubble's Law" *Lecture-Tutorial* on the accelerating expansion of the universe.

ready, at this stage of their education, to work with current models of dark energy that attempt to explain the accelerating expansion of the universe, they can still achieve a meaningful connection with this fascinating topic from modern cosmology early on in their Phys 101 courses.

III. DARK MATTER IN SPIRAL GALAXIES

Just as the Hubble plot for an accelerating universe allows Phys 101 students to understand one piece of evidence for the existence of dark energy, the rotation curves of spiral galaxies provide one piece of evidence, accessible to Phys 101 students, for the existence of dark matter.[30] A rotation curve is a plot of the orbital velocities of objects in a galaxy as a function of their distances from the galaxy's center. To understand this piece of evidence, students must understand forces, Newton's laws of motion, and gravitation, in addition to kinematics. Rotation curves and dark matter thus provide instructors with an application of introductory dynamics that can be used to enrich a Phys 101 course.

In order to comprehend why rotation curves provide evidence for the existence of dark matter, students must first use the physics they have learned from their introductory dynamics lessons to understand what the rotation curves of spiral galaxies would look like in the absence of dark matter. (Note that in the following discussion, we assume students have already learned that a spherically symmetric gravitating object can be treated as if it is a point mass located at its center.) A cursory glance at an image of a spiral galaxy reveals two components that immediately stand out: a central bulge, whose surface brightness I scales with radius r (as measured from the galactic center) according to the de Vaucouleurs's law $I \sim \exp(-r^{1/4})$, and a disk, whose surface brightness decreases exponentially with increasing radius ($I \sim e^{-r}$).[31] Therefore, if the mass of a spiral galaxy were dominated by its luminous matter, matter should be preferentially concentrated in the galaxy's inner regions, where the surface brightness is highest. In the innermost regions (within 1 kpc for the Milky Way), we can approximate the density of matter ρ in the bulge as constant and roughly spherically symmetric. Phys 101 students can be led to use their knowledge of centripetal and gravitational forces to derive that the orbital velocities of objects within this region should scale as $v \sim r$. Outside this region, the mass inside an object's orbit should be roughly constant. Students can be led to use this information to derive that beyond the innermost regions of the galaxy the velocities of objects should scale as $v \sim r^{-1/2}$ (which is called *Keplerian* rotation and which describes the motion of planets orbiting the Sun).

Students must next be able to compare these expected rotation curves to observed rotation curves. In reality, as r becomes very large, v is approximately constant. It's important to help students understand that this discrepancy can be resolved if we abandon our assumption that we can treat the mass of the galaxy as concentrated in its center. Since the observed motions shown in the galaxy rotation curve are inconsistent with the distribution of visible matter, this implies that there must be much more mass in the galaxy than we can see. The fact that v is constant over large distances can be used to lead students through a derivation that the density of matter in a spiral galaxy should scale as $\rho \sim r^{-2}$.

The chain of reasoning necessary to infer the presence of dark matter from spiral galaxies' rotation curves is conceptually complex for students since they must understand the physics underlying both expected and observed rotation curves. Our national study of Astro 101 students found that lecturing alone is a highly inefficient way to teach this chain of reasoning to students. After explicit lecture-based instruction on rotation curves, but in the absence of research-validated activities specifically designed to help students construct this chain of reasoning for themselves, only between 20 and 40% of students in a given semester could even identify what a correct rotation curve would look like for spiral galaxies. Even fewer could explain why these rotation curves suggest the existence of dark matter.[20]

In order to help students construct this chain of reasoning for themselves, we created and validated the "Dark Matter" *Lecture-Tutorial*.[20] Figure 8 shows the first page of this *Lecture-Tutorial*. The questions in Fig. 8 help build students' intuitions about Keplerian rotation by having them examine a table of data about the Solar System. The questions that accompany this table help to direct students toward the idea that the Sun accounts for more than 99.8% of the mass of the Solar System, which, in turn, helps to explain why farther planets orbit the Sun slower than planets closer to the Sun. The *Lecture-Tutorial* then transitions to questions about the Milky Way Galaxy (abbreviated "MWG" in the *Lecture-Tutorial*). The first of these questions requires students to choose the shape for the rotation curve of the Milky Way, based on only its luminous matter. As expected, the majority of students initially provide answers consistent with Keplerian orbits for the outer regions of the Milky Way, just like the Solar System. The *Lecture-Tutorial* then presents students with the observed, flat rotation curve of the Milky Way, which provides an opportunity for cognitive conflict within the students' collaborative learning groups. The remainder of the *Lecture-Tutorial* is designed to help students resolve this conflict between their prediction for the motion of objects in the Milky Way based on luminous matter, and the actual observed motions. Figure 9 shows several of the questions that help students to resolve this conflict (note that Questions 8 and 9, which are referred to by these questions, are the questions that require students to choose a rotation curve for a MWG dominated by only luminous matter). Results from our national study of Astro 101 students illustrate that students who use the "Dark Matter" *Lecture-Tutorial* are better able to explain how spiral galaxies' rotation curves provide evidence for dark matter.[20]

Figures 8 and 9 both contain student debates. Student debates are fictionalized arguments between two or more students on a conceptually challenging point. Students working on a *Lecture-Tutorial* must determine if any of the fictionalized students are correct and why. These student debates act as valuable "course corrections" for students who have progressed through several questions with their naïve ideas intact. Student 1's explanation in the student debate in Question 15 in Fig. 9 encapsulates the conceptual reasoning we expect from students who complete this *Lecture-Tutorial*. We argue that a student who can articulate an argument like that of Student 1 understands, at a fundamental level, why spiral galaxies' rotations curves are evidence for the existence of dark matter.

During this process of concept building, we have found that many students and instructors are tempted to use the following line of reasoning: Since the velocity remains the same for objects at greater and greater distances from the center of a galaxy, there must be an increase in the net gravitational forces acting on these distant objects to keep them

An object's orbit depends on the "mass inside" its orbit (also known as the *interior mass*). For a planet in our Solar System, you can find the interior mass by adding the Sun's mass to the mass of each object between the Sun and the planet's orbit. For example, the interior mass to Earth's orbit would be the Sun's mass plus the mass of Mercury plus the mass of Venus.

Here is a table that lists each planet, the mass inside each planet's orbit, and the speed at which the planets orbit the Sun.

Planet	Interior Mass (solar masses)	Orbital Speed (km/s)
Mercury	1.00	47.9
Venus	1.0000027	35.0
Earth	1.0000057	29.8
Mars	1.0000060	24.1
Jupiter	1.00096	13.1
Saturn	1.0012	9.66
Uranus	1.0013	6.81
Neptune	1.0013	5.43

1) Where is the vast majority of mass in the solar system located? What object or objects account for most of this mass?

2) Two students are discussing their answers to Question 1:

Student 1: *I think the majority of the mass in the solar system must include both the Sun and the planets. As you get farther away from the Sun, the interior mass gets bigger and bigger because you include more planets.*

Student 2: *I disagree. The majority of the mass in the solar system is from just the Sun by itself. Sure the mass gets a little bigger as you include more planets, but the additional mass from planets is really small.*

Do you agree or disagree with either or both of the students? Explain your reasoning.

3) How do the orbital speeds of planets farther from the Sun compare to the orbital speeds of planets closer to the Sun?

Fig. 8. The questions from the first page of the "Dark Matter" *Lecture-Tutorial*. These questions help students to understand why the Solar System's rotation curve is Keplerian.

moving at these velocities at their farther distances. This is wrong. The net gravitational force still declines with distance, but not as fast as we expected when we assumed the galaxy's mass was concentrated toward its center and the orbits were Keplerian.

Phys 101 instructors who wish to extend their students' understandings of these concepts with more quantitative examples may consider problems such as the following:

(1) The Sun is located approximately 8 kpc from the center of the Milky Way Galaxy. Its orbital velocity is measured to be 220 km/s. How much mass lies inside the Sun's orbit?
(2) If there were no dark matter in the Milky Way Galaxy, how fast would an HII complex orbit the center of the Milky Way if it is three times as far from the center as the Sun?
(3) If the HII complex orbits the Milky Way Galaxy at the same orbital speed as the Sun, how much more mass lies within the HII complex's orbit than one would have expected in the absence of dark matter?

Questions such as these require students to make use of the derived dynamics relations described earlier between orbital velocities, amount of mass interior to an orbit, and distances.

Activities on rotation curves and dark matter, such as those discussed in this section, can elevate the intellectual engagement of a traditional physics lesson with an accessible and interesting example from modern astrophysics. Rotation curves and dark matter provide a real-world example of when an object—in this case, a galaxy—cannot simply be treated as a point mass located at the orbital center. This context for teaching Phys 101 content can address students' understandings of Newton's second law, circular orbits, and Newton's law of gravitation—all in a context that is intrinsically interesting and challenging to naïve ideas, and which invites students to learn about one of the great discoveries of modern science: dark matter.

IV. DISCUSSION

The mission of those organizing and participating in the Gordon Conference on "Astronomy's Discoveries and Physics Education" is to enliven physics instruction by incorporating examples drawn from our current astrophysical

11) Describe how the stars' orbital speeds shown in the real rotation curve for the MWG are different from the orbital speeds shown in the rotation curve you chose in Question 9.

12) Using the real rotation curve for the MWG, provide a new ranking for the orbital speeds of Star A, Star B, and the Sun, from fastest to slowest. Describe any differences between this ranking and the one you provided in Question 8.

13) Based on your answers to question 12, would you say that most of the mass of the Milky Way Galaxy is located at its center (as is the case with our Solar System)? Explain your reasoning.

14) Based on the MWG's real rotation curve and your answers to Questions 11-13, is the gravitational force felt by the MWG's stars greater than, less than, or about the same as what you expected from questions 8 and 9? Explain your reasoning.

15) Two students are debating their answers to the previous questions:

Student 1: *Stars far from the center of the Milky Way are all moving at about the same speed. If most of the Milky Way's mass is located in its center, then stars far away from the center would orbit slower than stars closer to the center. Since this is not what we see, this must mean there is more mass throughout the outer regions of the galaxy than we can see. This also means that the Milky Way's stars feel a greater gravitational force than we originally expected.*

Student 2: *I disagree. There are fewer stars in the outskirts of the Milky Way than in the center, so there's less mass out there than at the center. Most of the Milky Way's mass must be at its center. So, since the stars are all going about the same speed, where the mass is located must not affect their speed. The gravitational force these stars feel probably gets weaker just like we would expect.*

Do you agree or disagree with either or both of the students? Explain your reasoning.

Fig. 9. A sample of questions from a single page of the "Dark Matter" *Lecture-Tutorial*. These questions help students to understand why a spiral galaxy's rotation curve implies the existence of an unseen mass component.

understanding of the universe around us.[1] We argue that some of those examples should reflect the discoveries of both dark matter and dark energy. These discoveries show that our understanding of the atom—which has driven many advances in physics and technology over the past century—pales in comparison to our ignorance about the matter and energy that comprise the overwhelming majority of the universe. In this paper, we discussed how instructors can integrate investigations of Hubble's law, Hubble plots, galaxy rotation curves, and dark matter and dark energy into the standard Phys 101 curriculum. We provided multiple examples of research-supported activities that instructors can use to simultaneously teach traditional Phys 101 content (such as graph interpretation, kinematics, and dynamics) and connect students with some of the most profound discoveries to emerge from modern cosmology.

We are not arguing that traditional problems on blocks, inclined planes, and pulleys need to be completely eliminated from the Phys 101 curriculum. They still serve valuable pedagogical purposes. For example, Redish discusses the importance of what he calls "touchstone" problems: These are problems that may not be intrinsically interesting in and of themselves, but still illustrate important physics principles and become the analogs upon which students build more sophisticated understandings of physics.[14] That being said, we still believe it is critical that we work to enliven the standard curriculum with contexts that reflect the real problems future generations of physicists will be involved in solving. Investigating cosmology provides a great topical vehicle to engage young STEM majors regarding the frontiers of scientific understanding—something often missing in the traditional introductory curriculum.

But before instructors rush to introduce some of the interactive learning activities we discussed in this paper, we have a word of caution. Research shows that while classes that use interactive learning activities typically do better than classes that do not, the average learning gains reported by interactive engagement classes have a very wide range.[32,33] Prior studies show that factors such as institution type, class size, gender, ethnicity, prior math and science coursework, GPA, and primary language may all be of secondary importance compared to how instructors implement research-validated instructional strategies in their classrooms.[32–34] Implementation really does matter. Instructors who are unfamiliar with the use of the activities discussed in this paper should consult the original research papers discussing their effectiveness and the additional references provided which detail implementation best practices (e.g., Refs. 13, 14, 17, 19, and 35).

However, effective implementation is not something that can be completely learned simply by reading a single article. It requires a serious and deliberate investment of an instructor's time and resources into his or her professional development. Thus, while we have provided some guidance in this paper on how to integrate these activities into a Phys 101 course, our discussion is unavoidably incomplete; our written text cannot give instructors the hands-on experiences they need in order to effectively implement these cosmology activities into their existing courses. It is our hope that instructors who want to infuse their Phys 101 curricula with astrophysical content and who want to teach using interactive engagement activities will invest in professional development experiences specifically designed to help instructors with their implementations.[36,37]

ACKNOWLEDGMENTS

Gina Brissenden provided valuable feedback on earlier versions of this manuscript. This manuscript also benefited from the multiple suggestions of its anonymous reviewers. This material is based in part upon work supported by the National Science Foundation under Grant Nos. 0833364 and 0715517, a CCLI Phase III Grant for the Collaboration of Astronomy Teaching Scholars (CATS). Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

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[37]See, for example, the Teaching Excellence Workshops run by the Center for Astronomy Education (CAE) at <http://astronomy101.jpl.nasa.gov/workshops>.